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NASA TM X-72827

NASA TM X-72827

CONSERVATIVE VERSUS NONCONSERVATIVE DIFFERENCING: TRANSONIC STREAMLINE SHAPE EFFECTS

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February 1976

(NASA-TM-X-72827) CONSERVATIVE VERSUS NONCONSERVATIVE DIFFERENCING: TRANSONIC STREAMLINE SHAPE EFFECTS Interim Report (NASA) 15 p HC $3.50 CSCL 01A

N76-21154

Unclas

G3/02 21481

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LANGLEY RESEARCH CENTER, HAMPTON, VIRGINIA 23665

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM X-72827		

4. Title and Subtitle	5. Report Date
CONSERVATIVE VERSUS NONCONSERVATIVE DIFFERENCING: TRANSONIC STREAMLINE SHAPE EFFECTS	February 1976
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Perry A. Newman and Jerry C. South, Jr.	
	10. Work Unit No.
9. Performing Organization Name and Address	505-06-11-02
NASA-Langley Research Center Hampton, Virginia 23665	11. Contract or Grant No.
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	NASA Technical Memorandum
National Aeronautics and Space Administration Washington, D. C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Interim technical information release, subject to possible revision and/or later formal publication.

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Aerodynamics Transonic Flow Compressible Flow Internal Flow Wind-Tunnel Walls Interference Conservation form	Unclassified unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	13	$3.25

*For sale by the National Technical Information Service, Springfield, Virginia 22161

CONSERVATIVE VERSUS NONCONSERVATIVE DIFFERENCING:

TRANSONIC STREAMLINE SHAPE EFFECTS

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Langley Research Center

SUMMARY

Streamline patterns calculated from transonic flow solutions which were generated using a nonconservative finite difference scheme show a net streamtube area increase far downstream of the disturbance indicating that the global mass balance has been destroyed. Similar calculations using a conservative finite difference scheme do not show this defect. Comparative calculations have been made at several freestream Mach numbers for nonlifting flow over a 10% parabolic arc airfoil. In a transonic internal (or confined) flow, this nonconservation of mass may be of greater concern than in an unconfined external flow.

INTRODUCTION

The basic relaxation technique introduced by Murman and Cole (ref. 1) has been used extensively in computing transonic flow fields. A deficiency was later recognized in the original finite difference scheme and Murman (ref. 2) introduced a conservative finite difference scheme, which included a shock-point operator, in order to correct it. Nevertheless, the original nonconservative scheme continues to be used by most investigators since it seems to give shock jumps and locations at the surface of the configuration more nearly like those observed in experiments. The purpose of this note is to demonstrate that these nonconservative shocks, which may extend well into the flow field, destroy the global mass balance by producing mass at the shock. In a transonic internal (or confined) flow this lack of mass balance may prove to be more crucial than is the case for an unconfined external flow.

The present results and observations were prompted by streamtube anomalies first noted in calculations pertaining to wind tunnel flows in both 2 and 3 dimensions. The influence of conservative versus nonconservative finite difference formulation on the global mass balance is readily observed by computing streamline shapes in an external 2-D transonic flow. This note presents

such sample results.

PROBLEM DESCRIPTION

A NASA Langley study of one concept for minimizing wind tunnel interference involves contouring the upper and lower (initially solid) walls of a small 2-D facility according to numerical results obtained from nonlinear flow solutions. Figure 1 shows calculated free-air streamline deflections (from straight lines) at the proposed tunnel-wall locations for 2-D subsonic lifting flow ($M = 0.5$, $\alpha = 3^{\circ}$) about a 10% parabolic arc airfoil. Arrows along the abcissa denote the test section and airfoil chord lengths in the streamwise direction. In this figure the deflections have been taken to be zero at the front end of the variable wall test section*. The point to be made here is that a streamtube roughly the dimensions of the tunnel returns to its far-upstream size far downstream of the model. That is, there is a global mass balance in the calculation. Figure 2 shows the same kind of results for a supercritical lifting flow ($M = 0.8$, $\alpha = 3^{\circ}$). In this case, however, the size of the streamtube has increased from far upstream to far downstream indicating that mass has been introduced. These results were generated using a program which employed the Garabedian and Korn (ref. 3) nonconservative finite difference scheme. In the $M = 0.5$ case there is no shock wave in the flow whereas in the $M = 0.8$ case there is one on the upper surface of the airfoil and it extends about 3/4 of a chord length into the flow field.

A computer program recently developed by South and Brandt (ref. 4) contained the Murman (ref. 2) conservative finite difference scheme and was easily modified to use the Garabedian and Korn (ref. 3) nonconservative finite difference scheme. This program solves the transonic small disturbance equation for only symmetric flow but incorporates several iterative solution techniques. For the results presented here, the equally-spaced computational grid was analytically stretched so that the physical grid extended to infinity in both the streamwise and normal directions. Streamline shapes were obtained along several grid lines by a streamwise integration of the normal component of the perturbation velocity using a trapezoidal rule in the computational grid.

Comparisons of conservative and nonconservative results were made with all aspects of the computation identical except the finite differencing scheme.

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*Note that for an unconfined 2-D lifting flow there is an appreciable streamtube deflection from upstream infinity to the front of the test section. One can infer this from the finite streamline slopes at large $|X|$ in figures 1 and 2. This phenomena presents an operational problem for those intent on producing "interference-free" confined flow.

In all cases the calculations were iterated until the root-mean-square value of the true residual was of the order of the truncation error. Although the solution itself was observed to change somewhat for a given case when the analytical stretching was changed, it is felt that the relative streamline shape effects presented here indicate that a nonconservative finite difference scheme destroys the global mass balance in a supercritical flow calculation.

RESULTS

Comparison cases were run for a 10% parabolic arc airfoil at zero incidence for free stream Mach numbers of 0., 0.70, 0.84 and 0.95. These represent incompressible, subcritical, mild supercritical, and strong supercritical flow conditions respectively. The computational grid was 128 streamwise by 33 in the normal direction (the physical half-space, airfoil mean plane to infinity). Streamline deflections were computed along several grid lines for all cases; not all of them are shown in the figures.

The incompressible ($M = 0$) and subcritical ($M = 0.70$) results for pressure distributions and streamline deflections were identical for conservative and nonconservative finite differencing. In all cases the computed streamtubes returned to their proper size. Since the conservative and nonconservative schemes differ only at points where the flow is supersonic, the results were expected to agree.

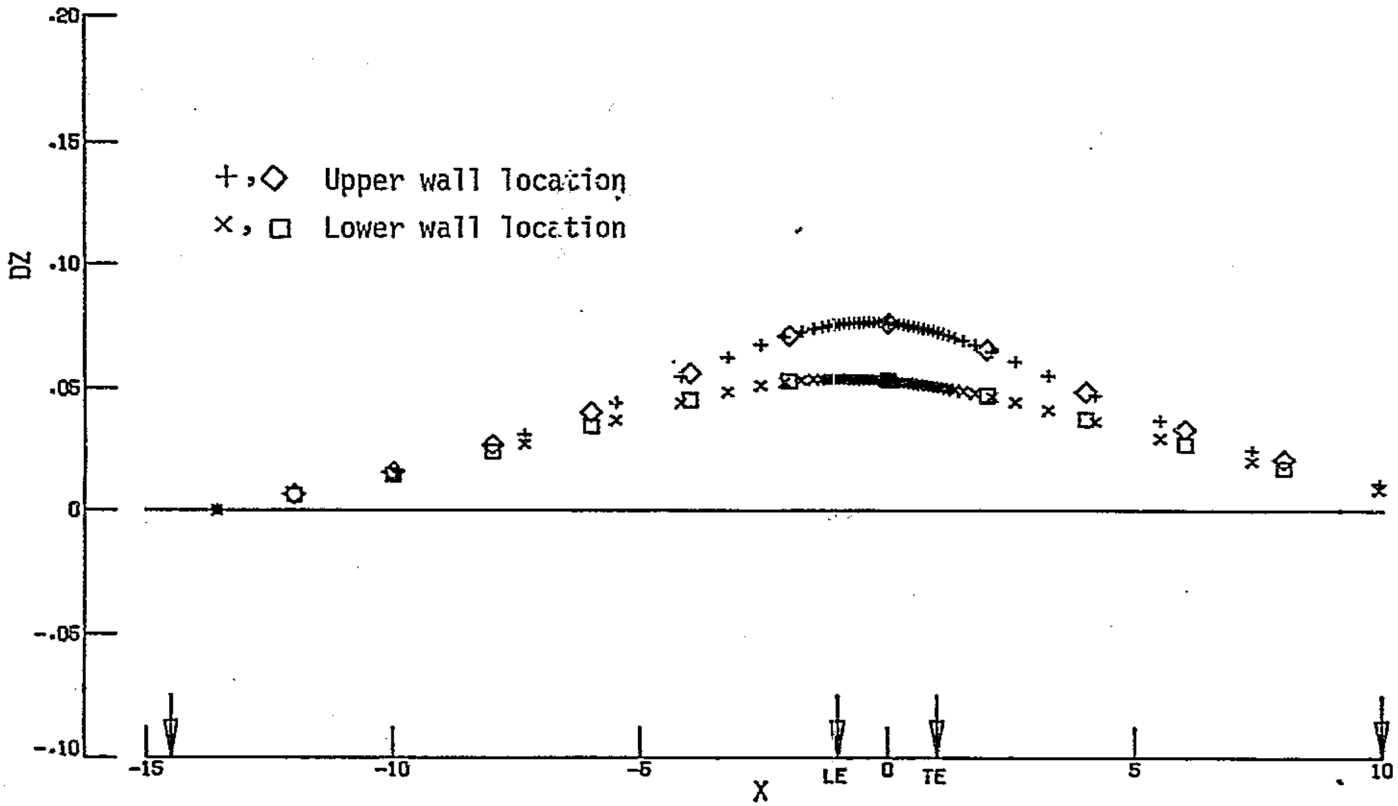
Results for the mild supercritical case ($M = 0.84$) are shown in figures 3 and 4. Figure 3 shows streamwise pressure coefficients along the symmetry line $y = 0$. Conservative results are given by the solid curves while nonconservative results are given by dashed curves. As others have shown in the past, one effect of the conservative scheme is to locate the shock wave further downstream on the airfoil surface. Figure 4 compares computed streamline deflections along three grid lines in the flow field. As can be seen from the figure, the displacements at downstream infinity indicate that the conservative streamtubes return to their upstream infinity value whereas the nonconservative ones do not. For this case, the shock wave extends about 1/2 a chord length into the flow.

Similar results for the strong supercritical case ($M = 0.95$) are shown in figures 5 and 6. In figure 5, which shows streamwise pressure coefficients along the symmetry line $y = 0$, it can be seen that the flows are no longer similar. In the nonconservative case there is a normal shock at the airfoil trailing edge whereas conservative differencing gives a weak oblique shock wave at the trailing edge followed by a normal shock about 1/2 chord length behind the airfoil. The streamline deflections for this case are shown in figure 6. Here it is very evident that the nonconservative streamline deflections do not become zero far downstream of the airfoil. On the other hand, however, the conservative streamtubes are seen to return to their proper size far downstream.

The extent to which the conservative and nonconservative flow fields differ in this strong supercritical case can be seen by comparing figures 7 and 8. Here Mach number contours in the supersonic bubble are shown for the nonconservative and conservative flows respectively. The multiple shock "fishtail" pattern shown in figure 8 is very similar to one computed by Murman (ref. 2) when he introduced his conservative scheme. The triangular region of nearly-constant, supersonic velocity between the weak oblique shock coming off the trailing edge and the shock wave further downstream is clearly evident. This downstream shock is normal at the symmetry line, becomes a strong oblique shock somewhat further out in the flow, and appears to again become normal after merging with the weak oblique shock coming off the airfoil trailing edge. The Mach number pattern shown in figure 7 was computed using the nonconservative scheme and has a normal shock coming off the trailing edge. The supersonic bubble is smaller than that for the conservative scheme and it appears to be terminated almost everywhere by a normal (or strong oblique) shock.

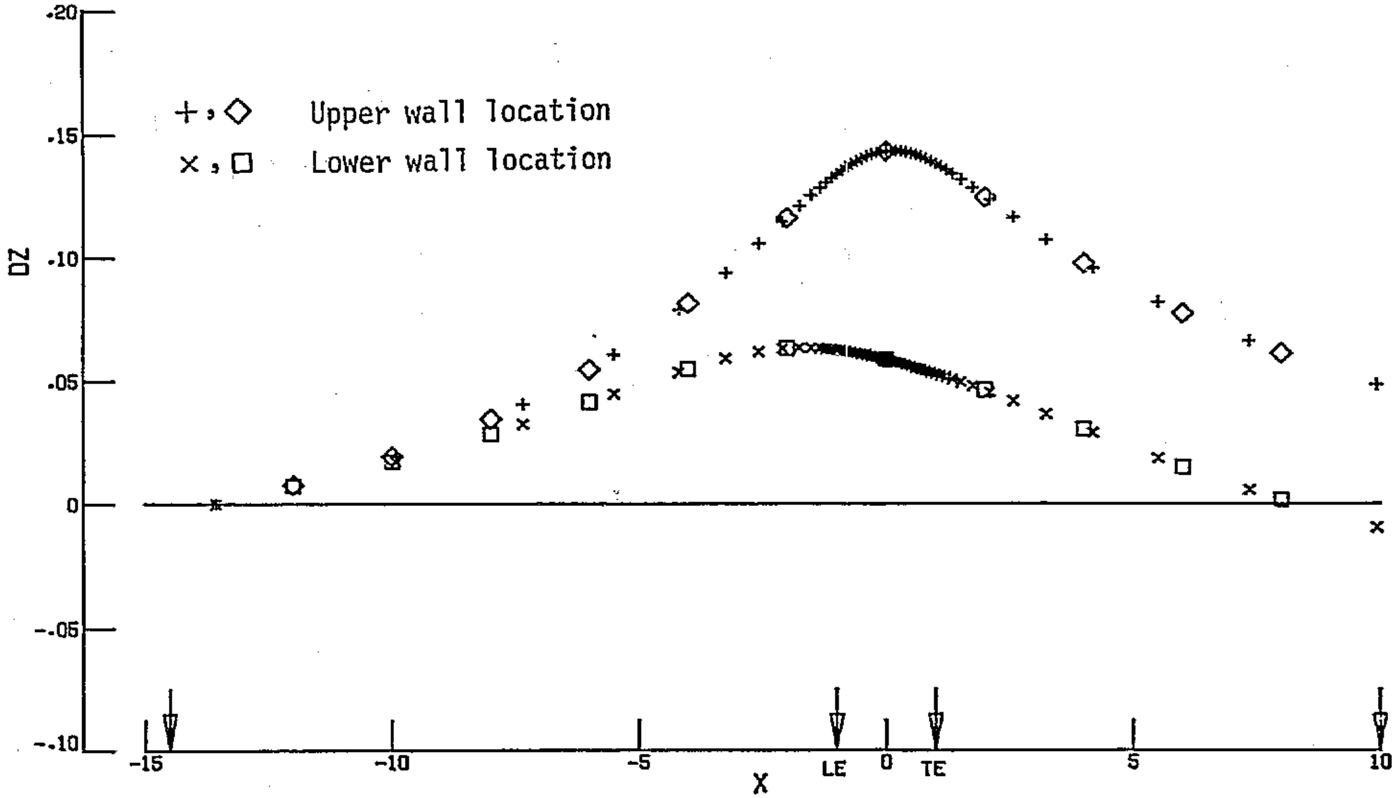
CONCLUDING REMARKS

Use of a nonconservative finite difference scheme in transonic flow calculations destroys the global mass balance. The mass created by the nonconservative operator at the shock produces a non-physical swelling of the inviscid streamtubes which persists far downstream. Perhaps the fortuitous agreement between the nonconservative and experimental results comes about because this streamtube swelling effect simulates a viscous wake or thickened boundary layer downstream of a shockwave. In any case, it is felt that the conservative finite difference scheme should be used in applications where streamtube effects are important, such as internal or confined flows.



10 PERCENT PARABOLIC ARC - 2D MODEL
APPROX. STREAM DEFLECTION DZ AT 6X19 UPPER AND LOWER TUNNEL WALL LOCATIONS
OPEN SYMBOLS - DEFLECTION AT TUNNEL WALL JACK POSITIONS
LINE SYMBOLS - DEFLECTION AT COMPUTATIONAL GRID POINTS
(MULTIPLY BOTH DZ AND X BY HCHORD= 2.0000 TO GET INCHES)

Figure 1.- Sample calculated wall deflections for subsonic lifting flow using nonconservative finite difference solution - no mass discrepancy.



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(MULTIPLY BOTH DZ AND X BY HCHORD= 2.0000 TO GET INCHES)

Figure 2.- Sample calculated wall deflections for supercritical lifting flow using nonconservative finite difference solution - pronounced mass discrepancy.

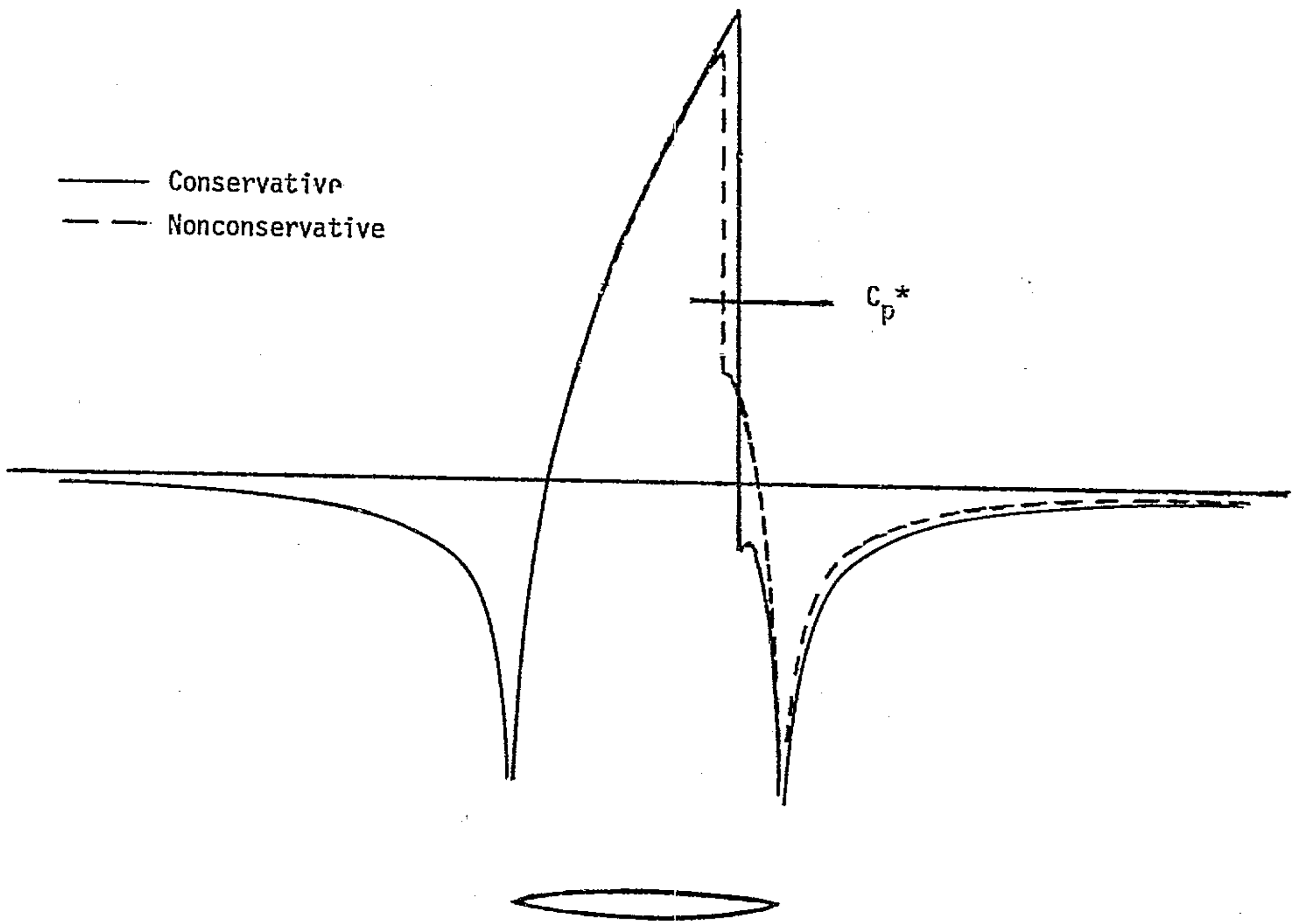

Figure 3.- Comparison of computed surface pressure coefficients for mild supercritical flow (M = 0.84) past a 10 percent parabolic arc airfoil at zero incidence.

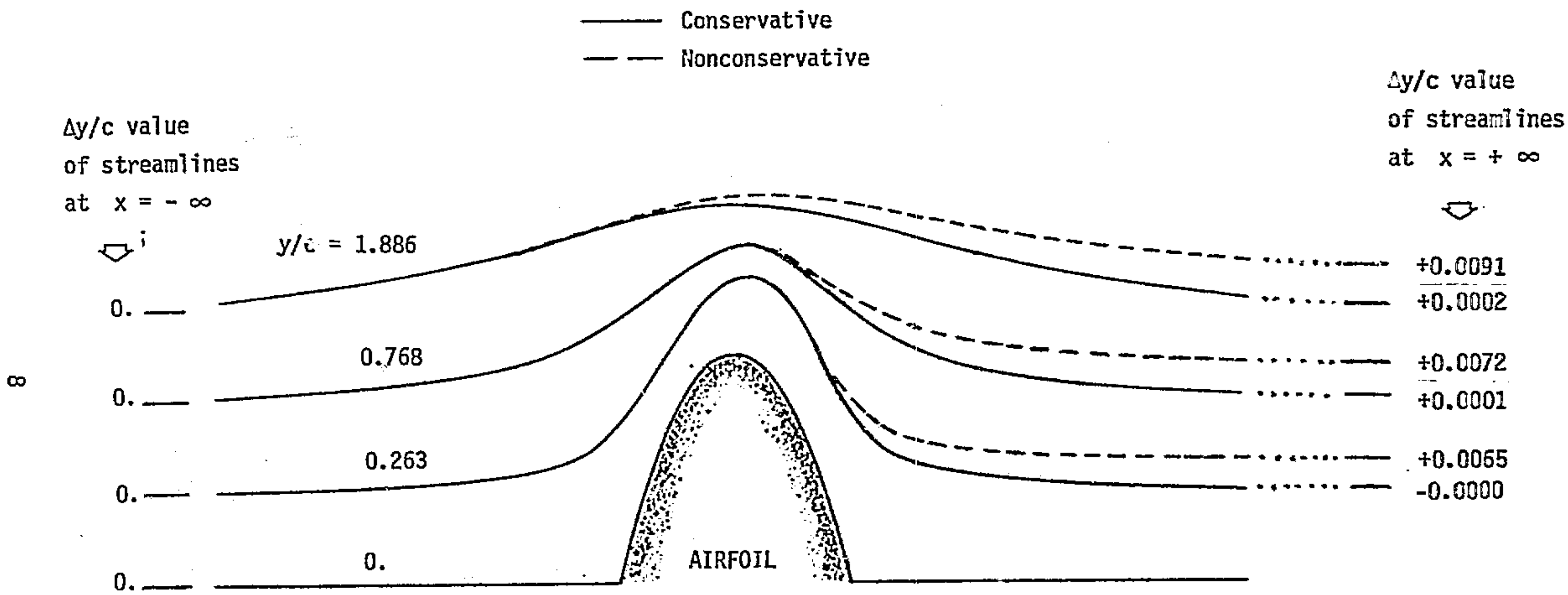

Figure 4.- Comparison of computed streamline deflections (Δy/c) for mild supercritical flow (M = 0.84) past a 10 percent parabolic arc airfoil at zero incidence.

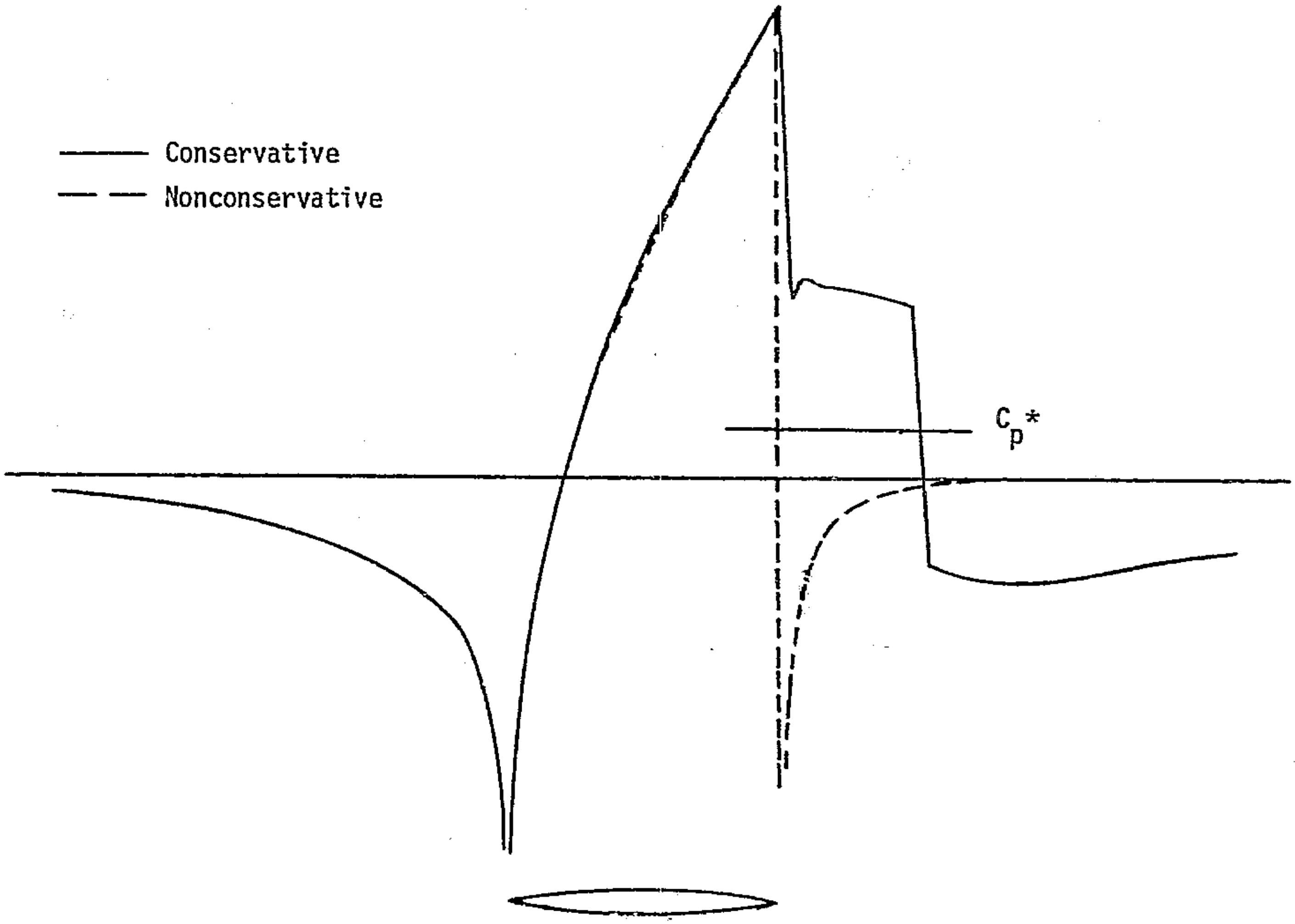

Figure 5.- Comparison of computed surface pressure coefficients for strong supercritical flow (M = 0.95) past a 10 percent parabolic arc airfoil at zero incidence.

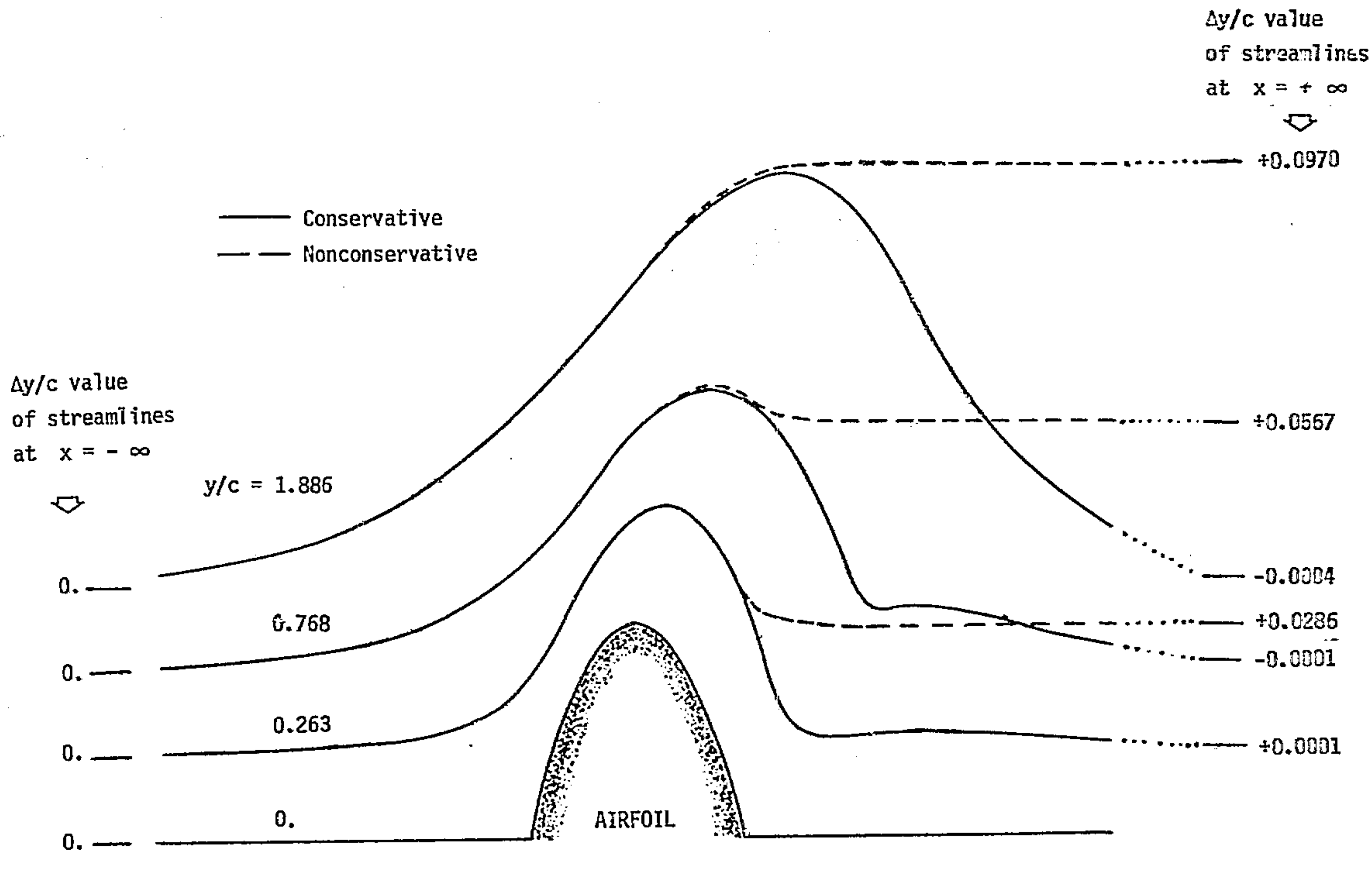

Figure 6.- Comparison of computed streamline deflections (Δy/c) for strong supercritical flow (M = 0.95) past a 10 percent parabolic arc airfoil at zero incidence.

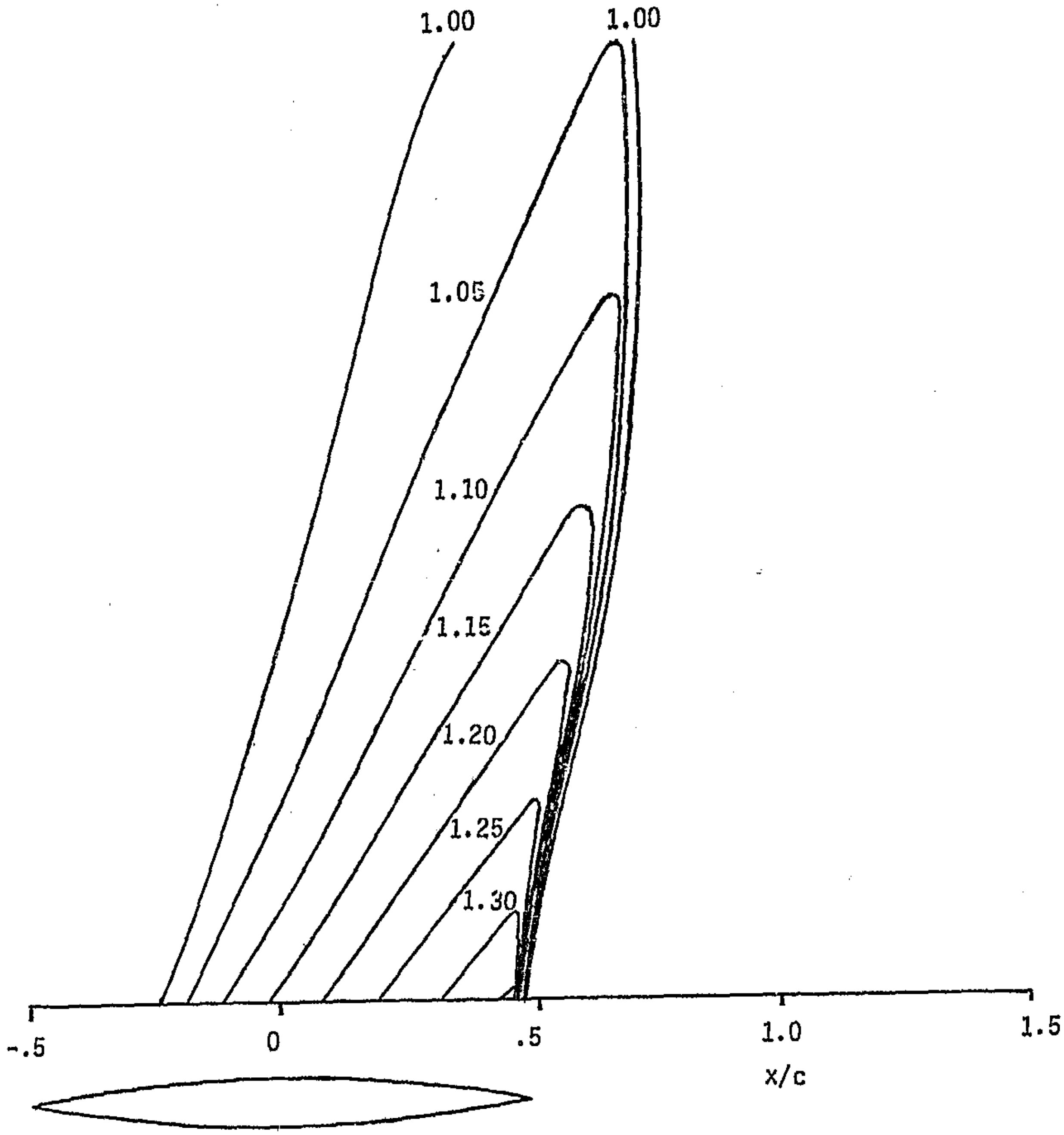

Figure 7.- Mach number contours in supersonic bubble of strong supercritical flow obtained using nonconservative finite differencing.

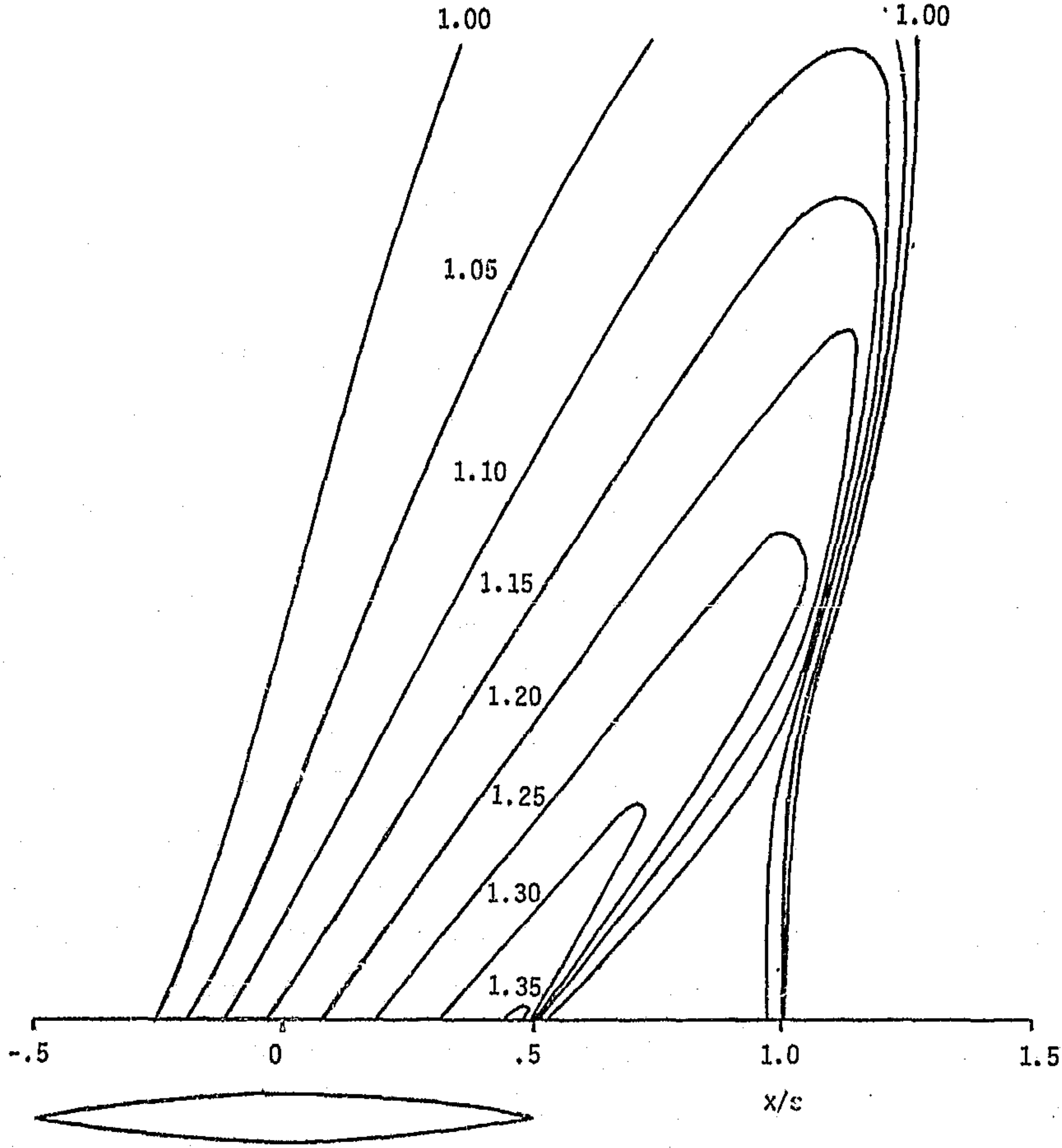

Figure 8.- Mach number contours in supersonic bubble of strong supercritical flow obtained using conservative finite differencing.

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